Temporal Accuracy of the Solution of Radiation Hydrodynamics Equations

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Traditional methods for time integration of the nonequilibrium radiation diffusion equations are first-order accurate in time, i.e., errors scale linearly with computational time step [1]. These methods rely on linearizations and operator splitting to simplify the linear algebra so small systems can be solved easily on the computer. A second more modern approach is to solve the nonlinear system in a single matrix that is large and complicated [2]. When employing this approach, it is fairly straightforward to make the time integration second-order (quadratic scaling of errors with time step) accurate in time but with increased computational cost. The purpose of this work is to investigate the middle ground between the first-order in time split and linearized methods and the second-order in time implicitly balanced methods. Recent works by the authors [3] and others have presented second-order in time split and linearized methods for nonequilibrium radiation diffusion.

The mathematical model for nonequilibrium radiation diffusion consists of two equations, one for conservation of energy in the radiation field and one for conservation of energy in the material field. If one couples the solution of the nonequilibrium radiation diffusion equations to a solution of the fluid motion in the material, the resulting system is a large, tightly coupled, system of nonlinear equations. Since we are only now beginning to have the computer capacity and the algorithmic efficiency to solve this as a single nonlinear system, traditional solution methods have had to rely on linearizations and operator splitting to make the solution tractable on existing computers with mainstream algorithms. It is important to analyze the trade-off between computational speed and accuracy when these simplifications (linearizing and operator splitting) are made.

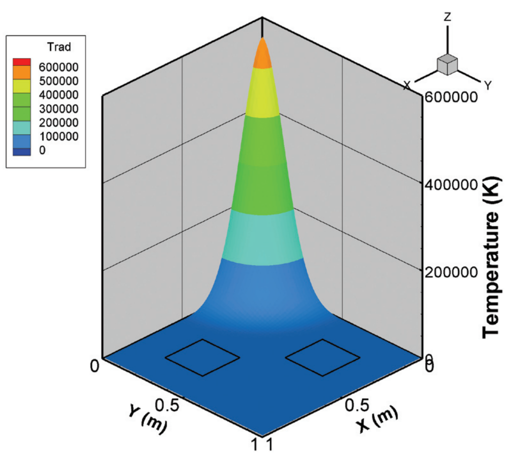

Fig. 1. ***Initial conditions for the 2-D Blast problem.***

Our benchmark, most accurate method for solving these equations is a popular modern scheme called the second-order Newton-Krylov method. This method is second-order in time, and contains no operator splitting and no linearizations. This nonlinear system of nonequilibrium radiation diffusion and material energy coupling equations is solved using Newton's method and the resulting linear equations of the Newton iterations are solved with a Krylov linear solver. This method is second-order accurate and robust but the solution of the large nonlinear system can be computationally expensive. Because of the potential complexity of the matrix equations to be solved, the efficiency of this method is heavily impacted by the preconditioning strategy.

This traditional workhorse algorithm [1] for solving the nonequilibrium radiation diffusion equations is first-order in time, operator split, and linearized. The basic idea of this approach is to recognize that if the material conduction effects are separated and solved on their own in an operator split step, then the material energy and the radiation energy can be combined, without introducing another split step, into a single matrix after the equations have been appropriately linearized. The solution of this matrix provides the new time radiation energy. The material conduction is then solved as a separate system, which yields the new time material temperature.

Our new contribution to second-order algorithms is motivated not only by previous studies of the behavior of the two algorithms just outlined, but also other attempts at developing linearized, and thus cheaper, second-order methods. During investigations of previously proposed second-order split and linearized algorithms, it was discovered that, for certain test problems, the separation of the diffusion and the reaction (coupling) terms into two distinct steps resulted in large and often fatal errors [4] which could cause the method to abort. These types of errors did not occur in the Newton-Krylov solution where there are no linearizations or operator splitting. However, the traditional solution method, although it had a large first-order in time error, ran these problems without difficulty. We have constructed a method that is second-order accurate in time that maintains the tight coupling between the radiation diffusion and the source coupling to the material energy. This new solution method uses what is known as Strang splitting to separate the material conduction from the material reaction and the radiation diffusion and reaction. We then replaced the first-order linearizations in the traditional linearized method of Olson with second-order linearizations.

An example of the test problems that we have used to assess these methods is presented below. An initial pulse of radiation energy is placed in the lower-left corner of the box with reflective walls over a radial extent of about 0.1 m. The two square obstacles have an atomic number of 10, giving those regions opacities 1000 times higher than that of the background material, which has an atomic number of one. The initial conditions are shown in Fig. 1. A contour plot of the resulting final radiation temperature at a time of 5 ns is shown in Fig. 2.

Most recently, we have added the hydrodynamic equations to our studies to model more realistic problems that include fluid flow, like radiation-driven shocks and blast waves in real fluids. Our results with the fully coupled Newton-Krylov methods, which solve the Euler equations of hydrodynamics with the radiation diffusion equations in a fully coupled manner, verify that we can generate second-order results, while an operator-split method very similar to that used in large production codes is only first-order in time. Our immediate goals are to (1) improve our prototyping to mimic a production code algorithms more closely, (2) confirm that we can generate both first-

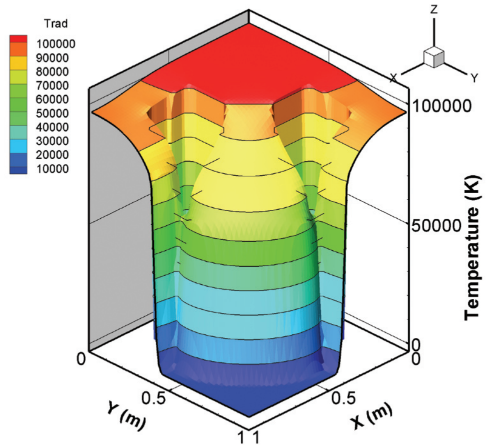

Fig. 2.
Radiation temperature contours for the 2-D Blast problem.

and second-order results when expected, and then (3) propose, if feasible, methods that display improved accuracy without incurring the cost of second-order Newton-Krylov.

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